



Кафедра електроніки, робототехніки і технологій моніторингу та інтернету речей
Факультет авіонавігації, електроніки та телекомунікацій (ФАЕТ)



Електронні системи

Electronic Systems

Lecture #13
Яновський, Фелікс Йосипович
 професор, доктор технічних наук,
 лауреат Державної премії України, IEEE Fellow

Орієнтовний тематичний план лекцій

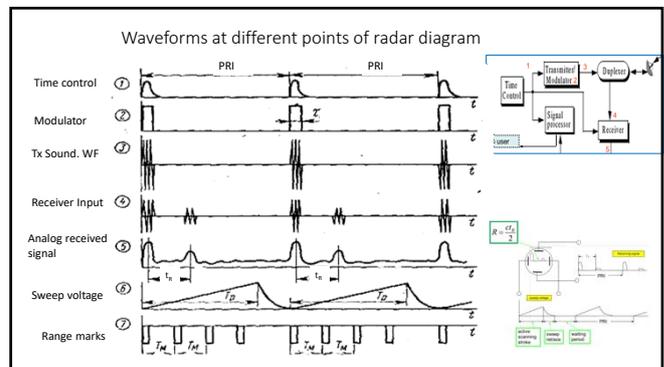
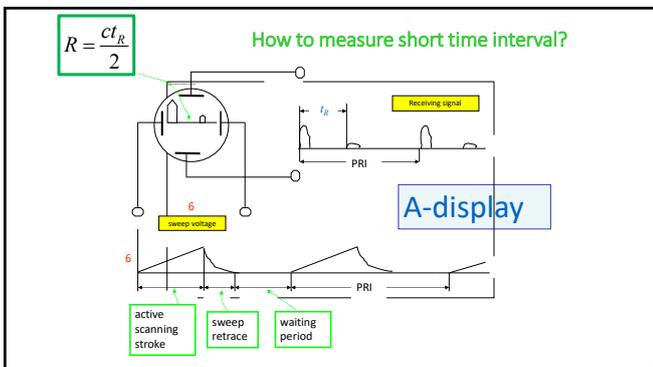
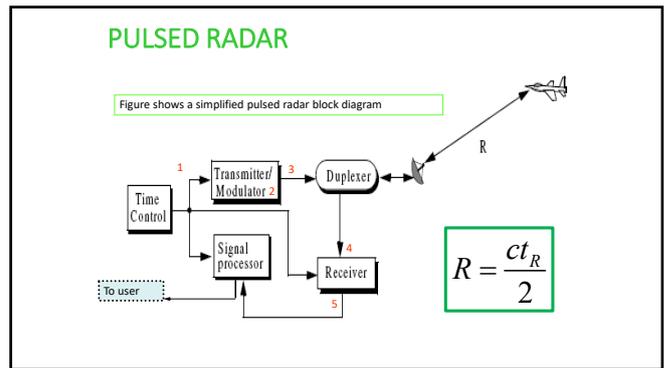
Основи теорії систем, сигнали і первинні перетворювачі електронних систем

1. Вступ. Визначення і термінологія, класифікація	2	
2. Характеристики електронних систем	2	
3. Теорія систем, аналіз електронних систем	2	
4. Первинні перетворювачі електронних систем	4	
5. Сигнали електронних систем	2	
6. Компоненти і обробка сигналів в ЕС	1	7 семестр
7. Експлуатаційні характеристики електронних систем	2	
8. Технічні характеристики електронних систем	2	
9. Технічна реалізація системи	1	
10. Електронні системи локації	18	
11. Електронні системи зв'язку	8	8 семестр
12. Електронні системи авіоніки	19	
Всього годин	63	

Електронні системи локації

1. Основні терміни, принцип дії, класифікація та застосування. 2
2. Відбиваючі властивості об'єктів. 2
3. Виявлення сигналів. 4
4. **Дальність дії локаційної системи.** 2
5. Роздільна здатність локаційної системи. 2
6. Вимірювання дальності та швидкості об'єктів. 2
7. Вимірювання кутових координат. 2
8. Методи підвищення роздільної здатності і точності вимірювань. 2

18



Train of pulses

In general, a pulsed radar transmits and receives a **train of pulses**.
 The Inter Pulse Period (IPP) is T , and the pulse width is τ .
 The IPP is often referred to as the Pulse Repetition Interval (PRI) or Pulse Repetition Period (PRP).
 The inverse of the PRI is the PRF, which is denoted by $f_r = 1/T$.

During each PRI the radar radiates energy only during shot time τ and listens for target returns for the rest of the PRI.
 The radar transmitting duty cycle (factor) d_t is defined as the ratio $d_t = \tau/T$.

Inverse value is (скважність) $Q = T/\tau$ (off-duty factor).

The radar average transmitted power is $P_{av} = P_t d_t$, where P_t denotes the radar peak transmitted power. The pulse energy is E_p .

$f_r = \frac{1}{PRI} = \frac{1}{T}$

$d_t = \tau / T$ Duty factor
 $Q = T / \tau$ Off-duty factor

$P_{av} = P_t d_t = P_t \tau / T = P_t \tau f_r$
 $E_p = P_t \tau = P_{av} T = P_{av} / f_r$

PULSED RADAR. Range Ambiguity

The range corresponding to the two-way time delay is known as the radar unambiguous range, R_u . Consider the case shown in Figure.

Echo 1 represents the radar return from a target at range $R_1 = c\Delta t/2$ due to pulse 1. Echo 2 could be interpreted as the return from the same target due to pulse 2, or it may be the return from a faraway target at range R_2 due to pulse 1 again. In this case

$R_2 = \frac{c\Delta t}{2}$ or $R_2 = \frac{c(T + \Delta t)}{2}$

PULSED RADAR. Range Ambiguity

Clearly, range ambiguity is associated with echo 2. Therefore, once a pulse is transmitted the radar must wait a sufficient length of time so that returns from targets at maximum range R_{max} are back before the next pulse is emitted. It follows that the maximum unambiguous range must correspond to half of the PRI

$T \geq \frac{2R_{max}}{C}$

$R_2 = \frac{c\Delta t}{2}$ or $R_2 = \frac{c(T + \Delta t)}{2}$

PULSED RADAR. Minimum range R_{min}

In case of sole antenna is used the value of Minimum Range of target detection is caused by the time duration, which is necessary to switch the antenna to receiver. Besides, the pulse duration also influences:

$R_{min} = \frac{c(\tau_u + t_o)}{2}$

τ_u — pulse length; t_o — time of inertia (deionization time in case of gas-discharge duplexer)

Radar Equation Description

The radar equation gives the received power level, P_{Rx} , of a radar signal after it has reflected off a target at some distance, R , from the radar.

The equation shows the dependence of P_{Rx} on radar and target parameters such as transmit power, radar cross section, etc. Finally it is used to calculate the maximum range of the radar R_{max} (energetic approach).

Радіолокаційне рівняння дає рівень потужності прийнятого сигналу P_{Rx} , відбитого від цілі на певній відстані, R , від радара.

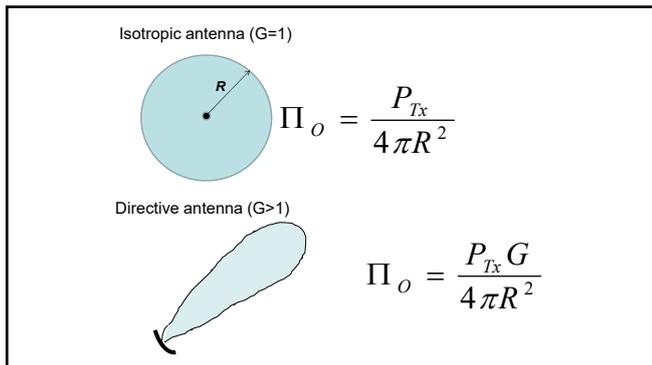
Рівняння показує залежність P_{Rx} від параметрів радіолокатора і цілі, таких як потужність передавача, ефективна площа розсіяння цілі σ тощо.

Нарешті, це рівняння використовується для обчислення максимальної дальності дії радіолокатора R_{max} (енергетичний підхід).

Radar Channel

Suppositions:
 Homogeneity of the Atmosphere;
 Absence of attenuation EM waves;
 Absence of the interferences;
 No influence of the Earth.

P_{Tx} - Tx power
 G - Antenna gain
 R - Range



Reflective properties of an object

- Radar range in perfect channel depends on reflective properties of the object.
- It depends on size, shape, material, wavelength, direction of observation, etc.
- To estimate total influence of all factors, a special parameter is introduced – RCS of the object.
Ефективна площа розсіяння (ЕПР) об'єкту σ .
- **Визначення ЕПР**

Reflecting properties of an object - RCS

- If $RCS = \sigma$, the object radiates power

$$P_{\text{Reflected}} = \Pi_o \sigma$$

- Then in receiving antenna at distance R from object, power density is created:

$$\Pi_{Rx} = \frac{P_{\text{Reflected}}}{4\pi R^2} = \frac{\Pi_o \sigma}{4\pi R^2}$$

- Substitute

$$\Pi_o = \frac{P_{Tx} G}{4\pi R^2}$$

$$\Pi_{\text{Reflected}} = \frac{P_{Tx} G \sigma}{(4\pi R^2)^2}$$

$$P_{Rx} = \Pi_{\text{Reflected}} A_e$$

$$P_{Rx} = \frac{P_{Tx} G A_e \sigma}{(4\pi)^2 R^4}$$

From this, it is easy to define the range of operation of a radar system in free space

Maximum range R_{max}

- If $P_{Rx} = P_{\text{threshold}} = S_{min}$
- $P_{\text{threshold}} = S_{min}$ – threshold power – min power of the useful signal at the receiver input that provides required quality of system operation (detection probability, accuracy, etc.)

$$R_{max} = \sqrt[4]{\frac{P_{Tx} G A_e \sigma}{(4\pi)^2 S_{min}}}$$

In case single antenna (Tx and Rx)

$$G = 4\pi \frac{A_e}{\lambda^2}$$

$$R_{max} = \sqrt[4]{\frac{P_{Tx} G^2 \lambda^2 \sigma}{(4\pi)^3 S_{min}}}$$

$$R_{max} = \sqrt[4]{\frac{P_{Tx} A_e^2 \sigma}{4\pi \lambda^2 S_{min}}}$$

Decomposed Radar Equation

$$P_{Rx} = P_{Tx} G \cdot \frac{1}{4\pi R^2} \cdot \sigma_{Target} \cdot \frac{1}{4\pi R^2} \cdot \frac{\lambda^2 G}{4\pi}$$

Power from radar ↙

Power density at target ↙

Power reflected by target to the radar ↙

Power density at radar ↙

Power collected by radar antenna (i.e., power received) ↙

Radar Equation

$$P_{Rx} = \frac{P_{Tx} G^2 \lambda^2 \sigma}{(4\pi)^3 R^4}$$

A) Without losses

B) For concentrated targets

What is concentrated target?

Threshold power – Порогова потужність

- Radar equation serves for determination of range of operation in both basic modes: Detection and Measurement of parameters.
- Depending on the mode and features of the task different value of threshold power (sensitivity) is required.
- Радіолокаційне рівняння служить для визначення дальності роботи в обох основних режимах: Виявлення цілі та Вимірювання її параметрів.
- Залежно від режиму та особливостей завдання може бути необхідним потрібне різне значення порогової потужності (чутливості).

Influence of losses, conditions EMW propagation, etc. Вплив втрат, особливостей поширення тощо

- Equation should be corrected according to real conditions

$$R_{max} = \sqrt[4]{\frac{P_{Tx} G A_e \sigma 10^{-0.2 \alpha_{km-at} R_{max}}}{(4\pi)^2 P_{thresh} L}}$$

$$R_{max} = R_0 10^{-0.05 \alpha_{km-at} R_{max}}$$

α_{km-at} [dB/km] – attenuation

Radar Equation without Losses

Restating P_{Rx} with σ substituted for RCS:

$$P_{Rx} = \frac{P_{Tx} G^2 \lambda^2 \sigma}{(4\pi)^3 R^4}$$

NO LOSSES WERE
TAKEN INTO
ACCOUNT !!!

- There are losses that need to be considered also.
1. The loss due to multipath (ground / building reflections) can lessen your P_{Rx} .
 2. Also, attenuation is given to the radiation by the atmosphere.
- These losses are designated L_{MP} and L_A , respectively.

Influence of multi pass & atmospheric losses

These (dimensionless) losses lessen the effectiveness of P_{Rx} , thus they divide P_{Rx} :

$$P_{Rx} = \frac{P_{Tx} G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 L_{MP} L_A}$$

This is the theoretical signal power measured at the antenna terminals.

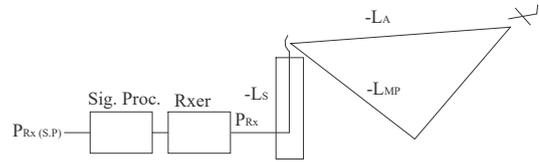
System Losses

3. If P_{Rx} is measured at the bottom of the tower (at the input of the receiver) then **transmission line losses** or system losses, L_S , need to be considered:

$$P_{Rx} = \frac{P_{Tx} G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 L_{MP} L_A L_S}$$

Losses

Losses (not considering space loss) and gains



In reality some more sources of losses exist, but we have considered the main losses.

Influence of interferences

Interfering power, P_I (not dimensionless) also needs to be considered. This lessens the effectiveness of your received power, thus it divides your P_{Rx} :

$$\frac{P_{Rx}}{P_I} = \frac{P_{Tx} G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 L_{MP} L_A L_S P_I}$$

This is also known as the Signal to Interference Ratio, or **S/I Ratio**

SIR

Thus:

$$\frac{S}{I} = \frac{P_{Rx}}{P_I} = \frac{P_{Tx} G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 L_{MP} L_A L_S P_I}$$

If we include in the definition of P_I any noise added by the receiver, then we can use that formula to give the power at the output of the Rxer:

$$\frac{S}{I} = \frac{P_{Rx, \text{at S.P. input}}}{P_I} = \frac{P_{Tx} G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 L_{MP} L_A L_S P_I}$$

Process Gain

The signal processor will increase the S/I ratio, or give it gain. This gain (dimensionless) is known as process gain, G_p . It increases S/I, and, thus, it is multiplied by the right side:

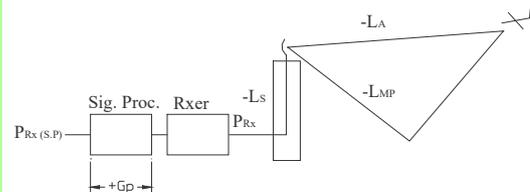
$$\frac{S}{I_{\text{At S.P. Output}}} = \frac{P_{Tx} G^2 \lambda^2 \sigma \cdot G_p}{(4\pi)^3 R^4 L_{MP} L_A L_S P_I}$$

This is the generic form of the radar equation.

If gain is given to the S/N by the receiver, it will be captured in the process gain, G_p .

Losses and Gains

Losses (not considering space loss) and gains



Maximum Radar Range

For simplicity we use radar equation without losses:

$$P_{Rx} = \frac{P_{Tx} G^2 \lambda^2 \sigma}{(4\pi)^3 R^4}$$

P_{Rx} is the total power delivered to the radar signal processor.

Let $S_{min} = P_{Rx, min}$ denote the minimum detectable signal power. It follows that the maximum radar range R_{max} is

$$R_{max} = \sqrt[4]{\frac{P_{Tx} G^2 \lambda^2 \sigma}{(4\pi)^3 S_{min}}}$$

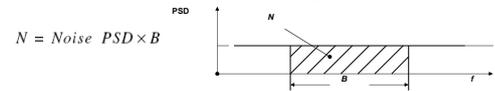
This equation suggests that in order to double the radar maximum range, one must increase the peak transmitted power sixteen times; or equivalently, one must increase the effective aperture four times.

Noise

In practical situations the returned signals received by the radar will be corrupted with noise, which introduces unwanted voltages at all radar frequencies.

Noise is random in nature and can be described by its Power Spectral Density (PSD) function.

The noise power N is a function of the radar operating bandwidth B .



The input noise power to a lossless antenna is $N_i = kT_e B$

$k = 1.38 \times 10^{-23}$ [Joule/degree Kelvin] is Boltzman's constant;

T_e is the effective noise temperature in [degree Kelvin].

Receiver Sensitivity

It is always desirable that the minimum detectable signal (S_{min}) be greater than the noise power. The fidelity of a radar receiver is normally described by a figure of merit called the noise factor F . $F = \frac{(SNR)_i}{(SNR)_o} = \frac{S_i/N_i}{S_o/N_o}$

S_i is the input signal power, N_i is the input noise power, S_o and N_o are, respectively, the output signal and noise power.

Substituting and rearranging terms yield $S_i = kT_e B F (SNR)_o$

Thus, the minimum detectable signal power can be written as

$$S_{min} = kT_e B F (SNR)_{o, min} \quad (SNR)_{o, min} \equiv m_{discr}$$

$$R_{max} = \left(\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 kT_e B F (SNR)_{o, min}} \right)^{1/4}$$

$$(SNR)_o = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 kT_e B F R^4}$$

Radar Equation

Radar losses denoted as L reduce the overall SNR, and hence

$$L = L_A + L_{MP} + L_S + L_{others}$$

$$(SNR)_o = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 kT_e B F L R^4}$$

$$R_{max}^4 = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 kT_e B F (SNR)_{o, min} L}$$

Although it may take on many different forms, this equation is what is widely known as the **Radar Equation**.

It is a common practice to perform calculations associated with the radar equation using decibel (dB) arithmetic

$$(X)_{dB} = 10 \log_{10}(X)$$

Radar Equation

$$R_{max}^4 = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 kT_e B F (SNR)_{o, min} L}$$

$P_N = kT_e B F$ is power of noise

$S_{min} = P_N (SNR)_{o, min}$ is sensitivity of receiver

Reminder what is $(SNR)_{o, min}$ – it is nothing but q_0

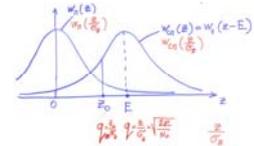
$$F = \frac{1}{2} [1 - \Phi(q_0)] \quad q_0 \text{ - threshold (relative threshold level)}$$

$$q = \frac{1}{\sigma_e} \sqrt{\frac{2E}{N_0}} \quad q \text{ - signal-to-noise ratio}$$

$$D = \frac{1}{2} [1 - \Phi(q_0 - q)]$$

$$q_0 = \frac{z_0}{\sigma_e} = \frac{\left(\frac{N_0}{2} \ln I_0 + \frac{E}{2}\right)}{\sqrt{\frac{N_0 E}{2}}} = \sqrt{\frac{N_0 E}{2}} \ln I_0 + \sqrt{\frac{N_0 E}{2}}$$

$$\sigma_e^2 = \frac{N_0 E}{2}$$



$$\Phi\left(\frac{z_0}{\sigma_e}\right) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{z_0}{\sigma_e}} \exp\left(-\frac{z^2}{2}\right) dz \text{ probability integral}$$

Radar Equation

$$R_{\max}^4 = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 k T_e F B (SNR)_{\min} L}$$

S_{min} - sensitivity

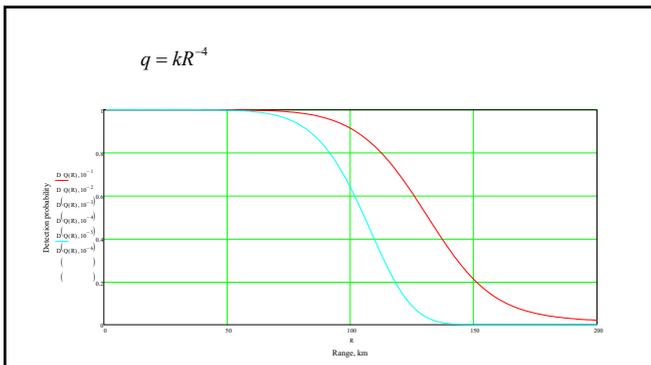
$P_N = k T_e F B$ is power of noise

$S_{\min} = P_N (SNR)_{\min}$ is sensitivity of receiver

m_{discr}

Range version of detection curves

- F and D are related $D = F \left(1 + \frac{q^2}{2}\right)^{-1}$
- where q is a parameter of detection, i.e. SNR for a single pulse.
- The farther the target the weaker the signal, i.e. $q = f(R)$
- For concentrated targets $E \propto R^{-4}$
- Expressing $q = f(R)$, one can come to range version of detection curves.
- From the curves one can clearly see the probabilistic nature of the notion R_{\max} .



It is a common practice to perform calculations associated with the radar equation using decibel (dB) arithmetic

$$(X)_{dB} = 10 \log_{10}(X)$$

Example: A certain C-band radar with the following parameters:

Peak power $P_t = 1.5 \text{ MW}$, operating frequency $f_0 = 5.6 \text{ GHz}$, antenna gain $G=45 \text{ dB}$, effective temperature $T_e = 290 \text{ K}$, pulse width $\tau = 0.2 \mu\text{sec}$. The radar threshold is $(SNR)_{\min} = 20 \text{ dB}$. Assume noise factor $F=3 \text{ dB}$ and target RCS $\sigma = 0.1 \text{ m}^2$. **Compute the maximum range.**

Solution: The radar bandwidth is $B = \frac{1}{\tau} = \frac{1}{0.2 \times 10^{-6}} = 5 \text{ MHz}$

The wavelength is $\lambda = \frac{c}{f_0} = \frac{3 \times 10^8}{5.6 \times 10^9} = 0.054 \text{ m}$

From Radar Equation: $(R^4)_{dB} = (P_t + G^2 + \lambda^2 + \sigma - (4\pi)^3 - kT_e F - (SNR)_{\min})_{dB}$

Before summing, the dB calculations are carried out for each of the individual parameters on the right-hand side. It follows:

$$R^4 = 61.761 + 90 - 25.352 - 10 - 32.976 + 136.987 - 3 - 20 = 197.420 \text{ dB}$$

$$R^4 = 10^{\frac{197.420}{10}} = 55.208 \times 10^{18} \text{ m}^4 \quad R = \sqrt[4]{55.208 \times 10^{18}} = 86.199 \text{ Km}$$

Thus, the maximum detection range is 86.2 Km

Propagation within the line-of-sight range

Height of ground-based antenna: h_1

Height of aircraft (target): h_2

Earth radius: $R_0 = 6371000 \text{ m}$

$AB = \sqrt{2R_0 h_1 + h_1^2}$

$BC = \sqrt{2R_0 h_2 + h_2^2}$

$R_0 \gg h_1, h_2$

$AB \approx \sqrt{2R_0 h_1}$

$BC \approx \sqrt{2R_0 h_2}$

$R = AB + BC = 3.57(h_1 + h_2)$

In case of $h_2 \gg h_1 \Rightarrow R \approx 3.57 h_2$ (the case air telecommunications)

Because of refraction $R_r = 4.12 \sqrt{h_2}$

If $h_2 = 1000 \text{ m}$, $R \approx 120\text{-}130 \text{ km}$ If $h_2 = 10000 \text{ m}$, $R \approx 350\text{-}400 \text{ km}$

- Thus, energetic detection range R_{max} determined with radar equation, is not reasonable to have more than the range of propagation within the line-of-sight R_r

$$R_{max} \leq R_r$$